Advanced Transmission Line Models for Real–Time Simulation of EM Transients in Power Systems.

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Introduction

Development of two Transmision Line (***TL***) Models for Simulation with a Multi–Processor Cluster.

Final Objective is to Produce Real–Time Versions.

TLs are Essential Components of Power Networks.

Power System Analysis and Simulation Requires Accurate ***TL*** Models.

Non Real–Time Models Available in ***EMTP***:

- **CP Line**: Accounts for Losses.
- **FD Line**: Includes most Frequency Dependencies.
- **WB Line**: Full Frequency Effects.

Introduction

FD Line Based Model:

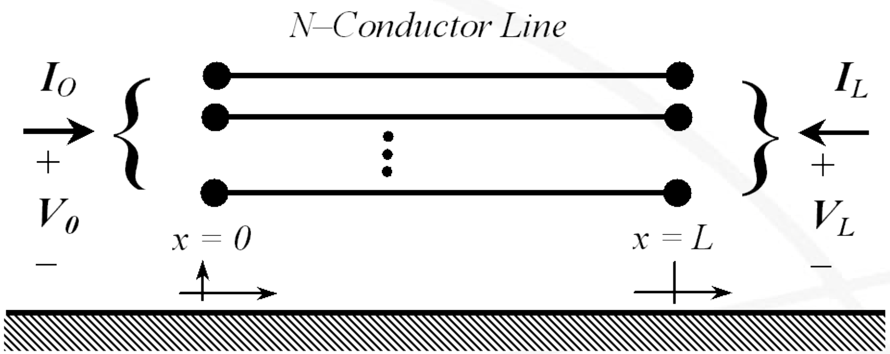
- Good Numerical Efficiency and Easy Application.
- Accurate for most cases of Aerial ***TLs***.
- Not Applicable to Cables

WB Line Based Model:

- Applicable to both Aerial ***TLs*** and Cables.
- 30–40 % More Computations than ***FD Line***.
- Certain Cases still Difficult to Apply.

Traveling Wave Line Models

- ***CP Line***
- ***FD Line***
- ***WB Line***



Far End ($x = L$) Reflected Current

$$I_{RL} = (I_L + Y_C V_L)/2$$

Arrives at $x=0$ as:

$$I_{AUX,0} = 2\, H \times I_{RL}$$

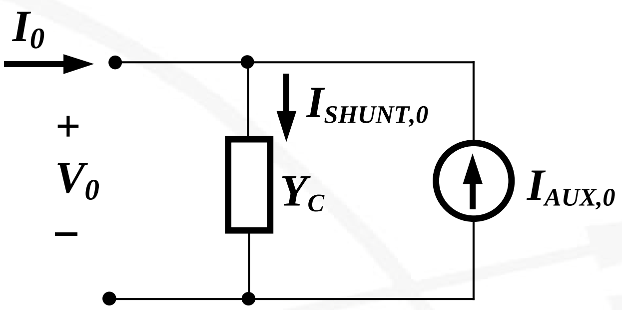
Transfer Function: $H = exp(-\Psi L)$, $\Psi = \sqrt{YZ}$

Traveling Wave Line Models

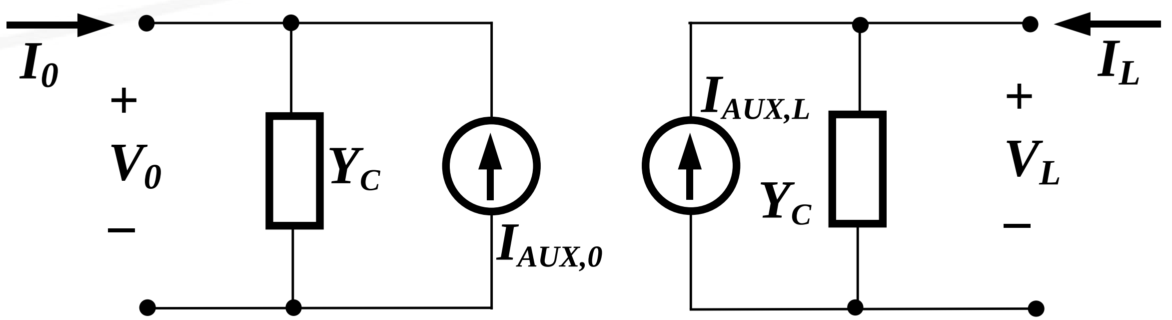
Currents at $x = 0$:

$$I_0 = Y_C V_0 - I_{AUX,0}$$

Model for Line End at $x = 0$



Complete *TW* Line model:



Terminals are decoupled in Time

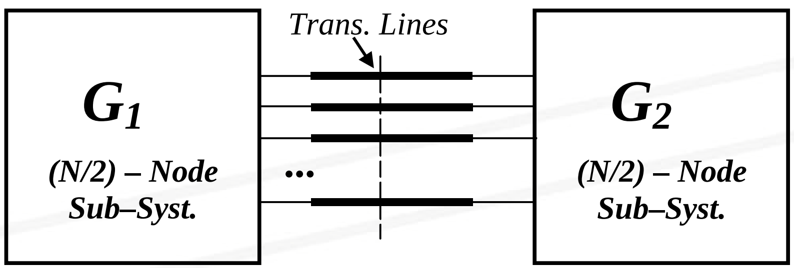
Network Partitioning

<u>*N*–node Network:</u>

Solution is order N^2
(Nr. of operations)

$$[G][V] = [I_S] + [I_{HIST}]$$

N–Node Sist.



Partition Through Lines:

$(N/2)^2+(N/2)^2 = N^2/2$
Operations.

Divide and Conquer!

Sparsity Decreases Operations too. <u>Nevertheless</u>, Partitioning facilitates Multi–Processor Paralleling.

$$\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{S1} \\ I_{S2} \end{bmatrix} + \begin{bmatrix} I_{H(1,1)}+I_{H(2,1)} \\ I_{H(2,2)}+I_{H(1,2)} \end{bmatrix}$$

From Frequency to Time Domain

Freq. Domain Model:

$$\boldsymbol{I_0 = Y_C V_0 - 2H I_{R,L}}$$

$$\boldsymbol{I_L = Y_C V_L - 2H I_{R,0}}$$

Time Domain Model:

$$\boldsymbol{i_0(t) = y_C(t) * v_0(t) - 2h(t) * i_{R,L}(t)}$$

$$\boldsymbol{i_L(t) = y_C(t) * v_L(t) - 2h(t) * i_{R,0}(t)}$$

•*Matrix–to–Vector Convolutions* ***.**

•*Convolutions are performed by Discrete–Time State–Space (***DSS***) Methods.*

State–Space Convolution.

FREQ. DOMAIN:

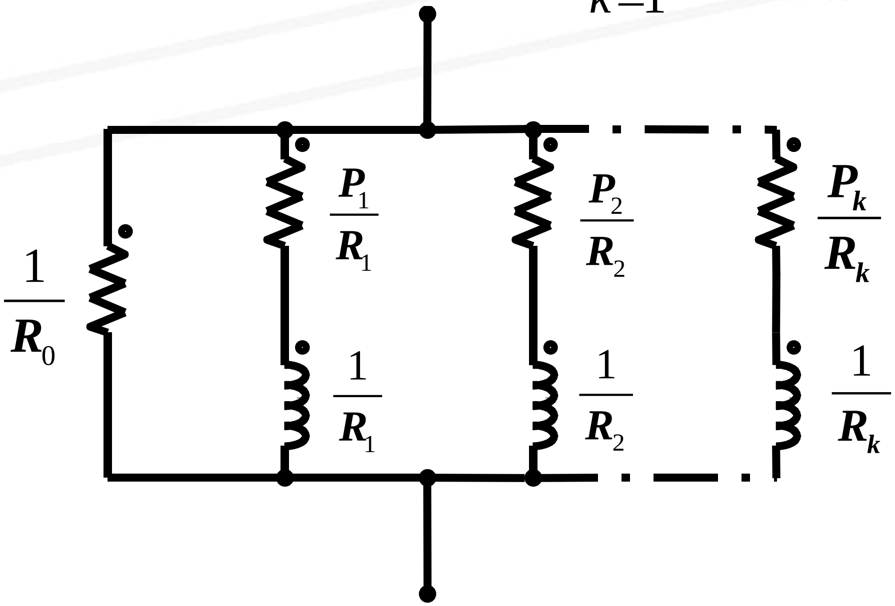
$$\boldsymbol{I} = \boldsymbol{Y}\boldsymbol{V}$$

TIME DOMAIN:

$$\mathbf{i}(t) = \boldsymbol{y} * \mathbf{v} = \int_0^t y(\tau)v(t-\tau)\,d\tau$$

RATIONAL FIT OF *Y*:

$$Y = R_0 + \sum_{k=1}^{N_{fit}} \frac{R_k}{s - p_k}$$



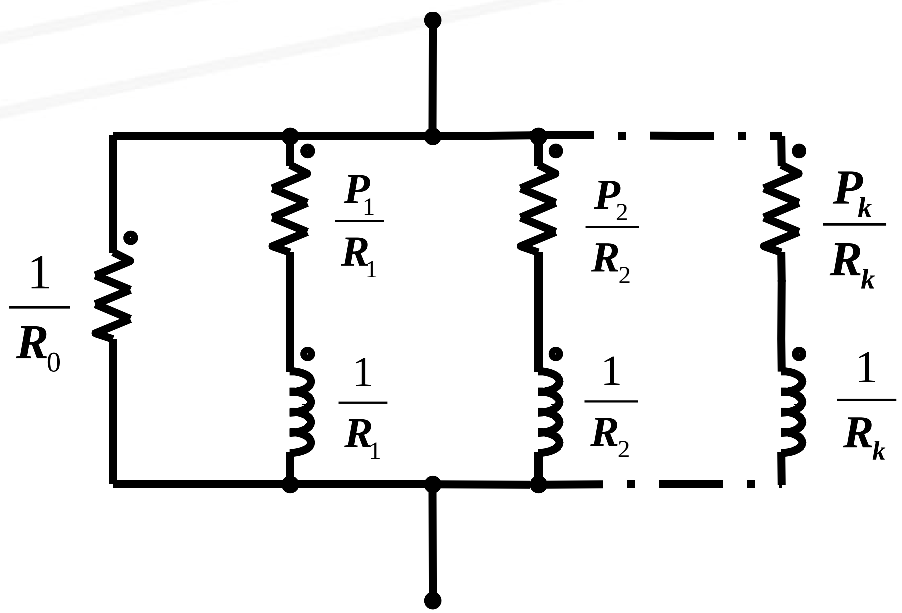
State–Space Convolution.

INTRODUCE N_{fit} STATE VARIABLES IN FREQ. DOMAIN:

$$I = X_1 + X_2 + \ldots + X_{Nfit} + R_0 V \qquad \textbf{... (1)}$$

WITH: $$X_k = \frac{R_k V}{s - p_k}, \quad k = 1,2,\cdots,N_{fit}$$

OR: $$(s - p_k) X_k = R_k V, \quad k = 1,2,\cdots,N_{fit} \qquad \textbf{... (2)}$$



Time Domain State–Space (SS).

TD FORM OF *(2)*:

$$
\begin{cases}
\dot{x}_1 = p_1 x_1 + R_1 v \\
\dot{x}_2 = p_2 x_2 + R_2 v \\
\quad\cdots \\
\dot{x}_{Nfit} = p_{Nfit} x_{Nfit} + R_{Nfit} v
\end{cases}
\quad \ldots (3)
$$

AND OF *(1)*:

$$
\mathbf{i}(t) = R_0 \mathbf{v}(t) + \sum_{k=1}^{Nfit} \mathbf{x}_k(t) \quad \ldots (4)
$$

(3) AND *(4)* ARE IN STATE SPACE (CONTINUOUS–TIME) FORM:

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{v} \\
\mathbf{i} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{v}
\end{aligned}
$$

Discrete–Time State–Space (DSS).

COMPUTER SOLUTION IS BY ***DSS***.

CT TO ***DT*** CONVERSION BY NUMERIC RULE: EULER, MID–POINT, GEAR, ETC.

MID–POINT EQUIV. TO TRAPEZOIDAL INTEGRATION:

$$y = \frac{dx}{dt} \qquad \Rightarrow \qquad \frac{y(n)+y(n-1)}{2} = \frac{x(n)-x(n-1)}{\Delta t}$$

DSS CONVOLUTION:

$$x_k(n) = a_k x_k(n-1) + b_k v(n-1),$$

$$k = 1,2,\cdots,N_{fit}$$

$$i(n) = R_0 v(n) + \sum_{k=1}^{Nfit} c_k x_k(n)$$

Convolution for Remote Currents.

H IS FACTORED IN A MINIMUM PHASE TRANSFER MATRIX $\tilde{H}$ AND A PURE DELAY TERM:

$$H = \tilde{H} e^{-s\tau}$$

$\tilde{H}$ IS THEN FITTED RATIONALLY:

$$\tilde{H} = \sum_{k=1}^{Nfit2} \frac{Q_k}{s - q_k}$$

FREQ. DOMAIN

$$I_k = \tilde{H} e^{-s\tau} I_R$$

TIME DOMAIN

$$i_{aux}(t) = \tilde{h}(t) * i_R(t - \tau)$$

- CONVOLUTION PROCEEDS NORMALLY WITH ***DSS***.
- DELAYED TERM $\boldsymbol{i_R(t-\tau)}$ REQUIRES INTERPOLATIONS.

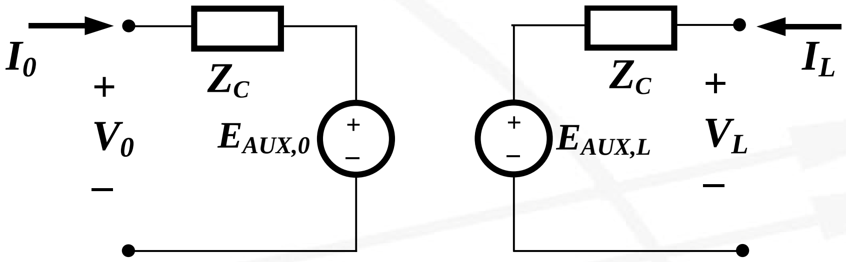
FD Line Model

1. TRANSFORM THE COUPLED ***N***–CONDUCTOR LINE TO MODAL DOMAIN.
2. PROBLEM NOW IS SOLVING ***N*** SINGLE–PHASE LINES (UNCOUPLED).
3. SOLVE IN MODAL DOMAIN AND TRANSFORM RESULTS BACK TO THE PHASE DOMAIN.
4. ASSUME MODAL TRANSFORMATIONS ARE CONSTANT AND REAL.
5. BECAUSE ASSUMPTION 4. MODEL IS ACCURATE ONLY FOR SYMMETRIC OR NEAR–SYMMETRIC AERIAL LINES.

Differences with Original FD Model

ORIGINAL MODEL DEVELOPED BY J. MARTI (1982):

1. THEVENIN EQUIVALENTS, Z_C AND VOLTAGE ***TWs***.
2. BODE FIT ONLY REAL POLES AND RESIDUES.



NEW VERSION:

1. NORTON EQUIVALENTS, Y_C AND CURRENT ***TWs***.
2. TAKES PARAMETERS FROM ORIGINAL FD (Z_C).
3. MODEL STRUCTURE FULLY BASED ON ***DSS***.
4. ACCEPTS COMPLEX POLES AND RESIDUES.
5. LOW ORDER FIT BY WEIGHTED VECTOR FITTING.
6. COMPUTER EFFICIENCY BY SOLVING COMPLEX STATES WITH REAL ARITHMETIC.
7. MODEL CAN BE SPLITTED TO ALLOW PARTITIONING OF LARGE NETWORKS.

Wide–Band (WB) Model.

1. BASIC CONCEPTS FROM UNIVERSAL LINE MODEL (***ULM***), DEVELOPED BY MORCHED, GUTAVSEN & TARTIBI (1998).
2. ACCOUNTS FOR FULL FREQUENCY DEPENDENCE EFFECTS.
3. APPLICABLE TO HIGHLY ASYMMETRIC LINES AND CABLES.
4. $\boldsymbol{Y_C}$ AND ***H*** MATRICES ARE FITTED IN PHASE DOMAIN.
5. ALL MATRIX ELEMENTS ADJUSTED WITH COMMON POLES.

IMPROVEMENTS:

1. LEANER CODE FULLY BASED ON ***DSS*** METHODOLOGY.
2. COMPLEX STATES RESOLVED WITH REAL ARITHMETIC.
3. FITTER BASED ON WEIGHTED VECTOR FITTING (***WVF***). MOST DIFFICULTIES WITH THE MODEL ARE CAUSED BY FITTERS.

Wide–Band (WB) Model.

FITTING OF $\boldsymbol{Y_C}$:

- ALL ELEMENTS FITTED WITH COMMON POLES.
- POLES ARE EXTRACTED FROM THE $\boldsymbol{Y_C}$ TRACE.

$$\boldsymbol{Y_C} = \boldsymbol{R_0} + \sum_{k=1}^{Ny} \frac{1}{s - p_k} \boldsymbol{R_k}$$

- $\boldsymbol{R_1}, \boldsymbol{R_2}, \dots, \boldsymbol{R_{Ny}}$ ARE MATRICES OF RESIDUES. $\boldsymbol{R_0}$ IS MATRIX OF RESIDUES AT $\omega = \infty$.

Wide–Band (WB) Model.

FITTING OF ***H*** :

- ***H*** IS FIRST DECOMPOSED IN MODAL DELAY TERMS.

$$\boldsymbol{H} = \sum_{k=1}^{N} \boldsymbol{H}_{\boldsymbol{k}}^{\boldsymbol{m}} e^{-s\tau_k}$$

- MODAL TERMS ARE GROUPED IF DELAY DIFFERENCES $\Delta\tau \leq 10°$.
- ALL ELEMENTS OF EACH GROUPED MATRIX ARE FITTED WITH COMMON POLES.
- FOR EACH GROUP, POLES ARE EXTRACTED FROM THE SUM OF EXPONENTIAL FUNCTIONS OF MEMBER MODES IN THE GROUP.

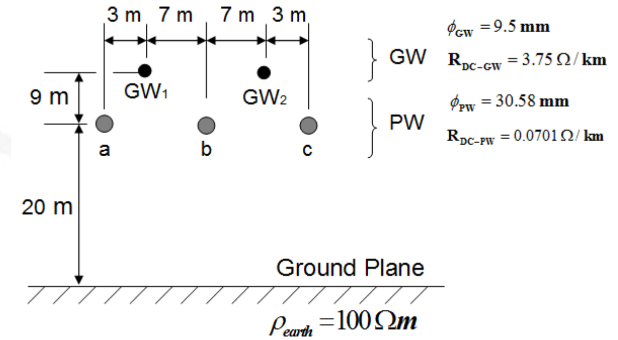
$$\boldsymbol{H} = \sum_{k=1}^{Ng} e^{-s\tau_k} \sum_{n}^{N_k} \frac{1}{s - q_{k,n}} \boldsymbol{Q}_{\boldsymbol{k,n}}$$

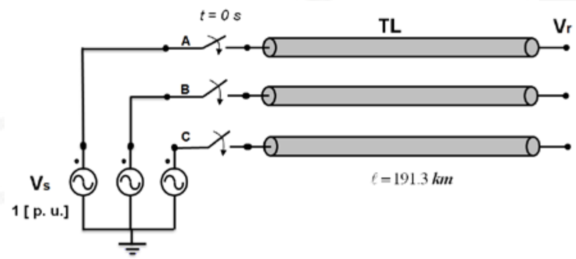
Example 1. FD Line

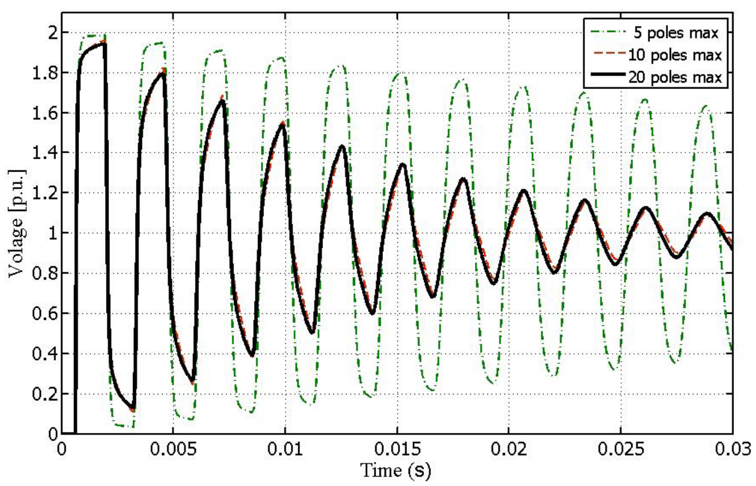
Phase A energizing with Unit Step. Voltage output at far –end A.

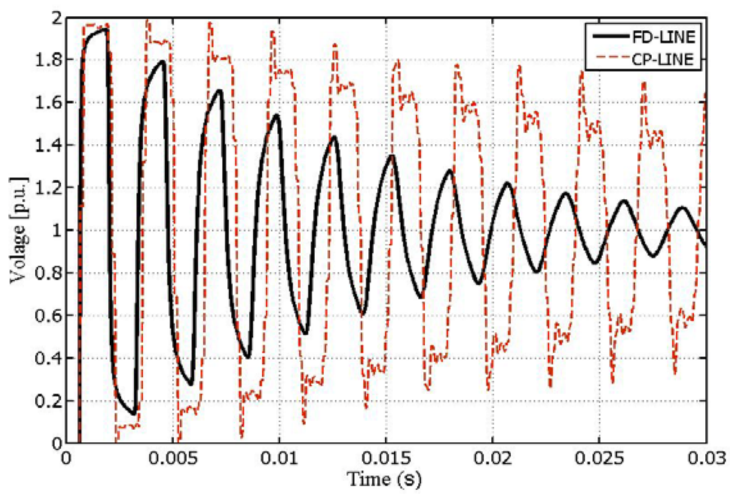
Figure 1. Real poles obtained with Bode Fit. Good accuracy with 20 poles. Less than 10 poles gives poor accuracy.

Figure 2. Four real and complex poles obtained with ***VF***, very good accuracy. Comparison with ***CP Line*** Model (red line).









Example 2: FD Line.

Same line data as in Example 1.

Source Impedance is series ***R–L***, ***R=10^{-3} Ω***, ***L= 0.05 H***.

Load Impedance is 3 series *RLs* to ground, ***R= 500 Ω***, ***L****=0.02 H*.

Simultaneous Energizing at t=0 s. Fault occurring at mid–line on phase C at ***t=15 ms***.

Phase C is disconnected from source at ***t=65 ms***.

Figure 1 is sending–end voltages with ***TPs*** at line side.

Figure 2 is sending –end currents with ***TCs*** at source side.

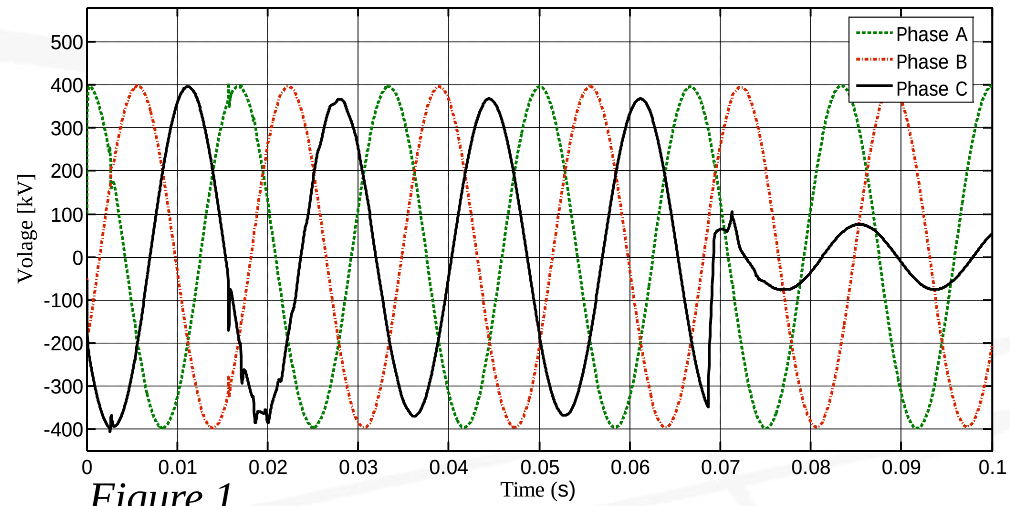


Figure 1

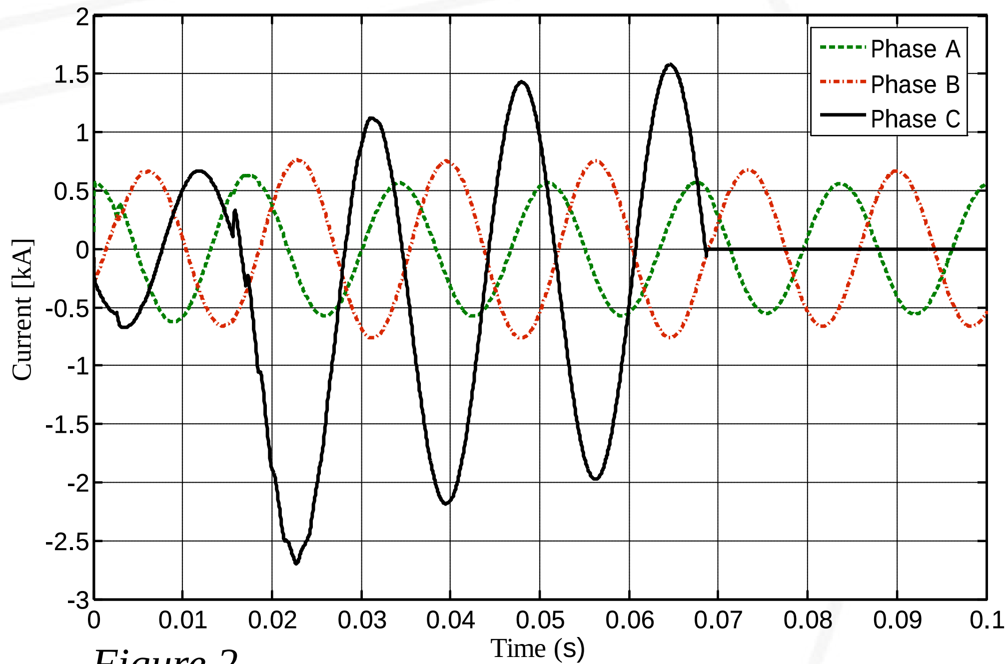


Figure 2

Example 2: FD Line (Continuation).

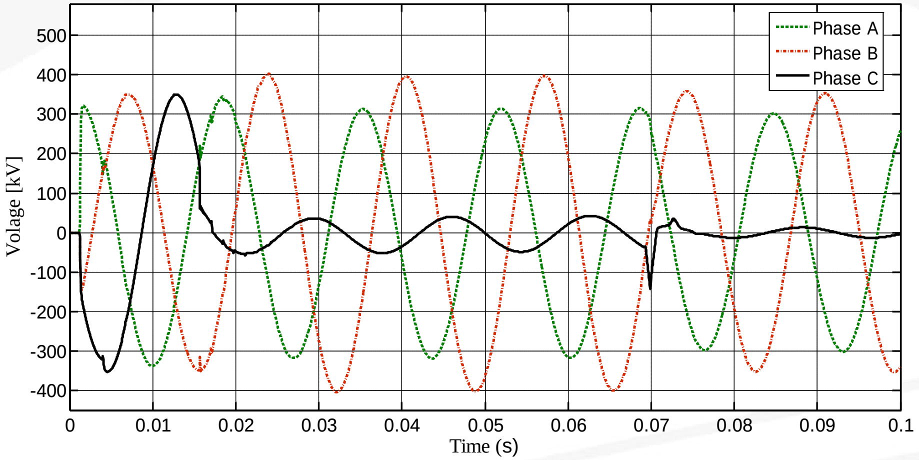


Figure 3. Voltage waveforms at load line–end.

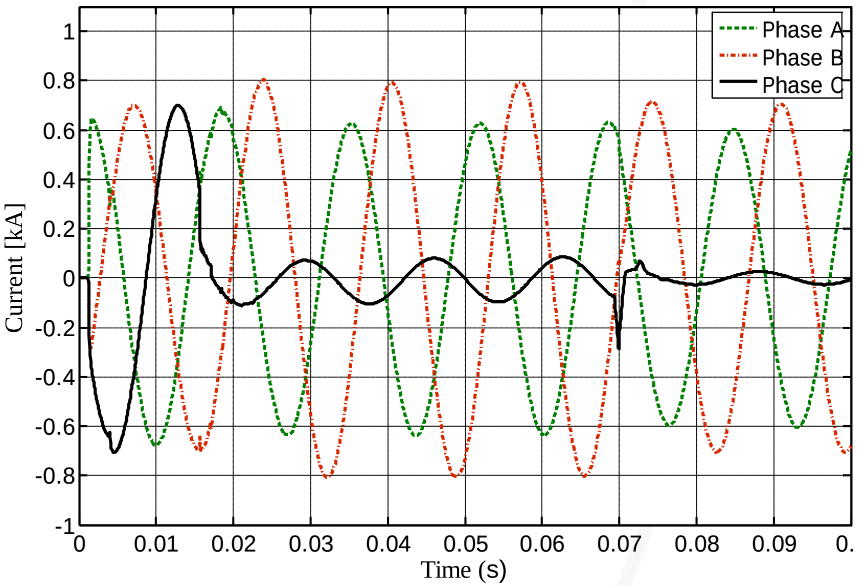
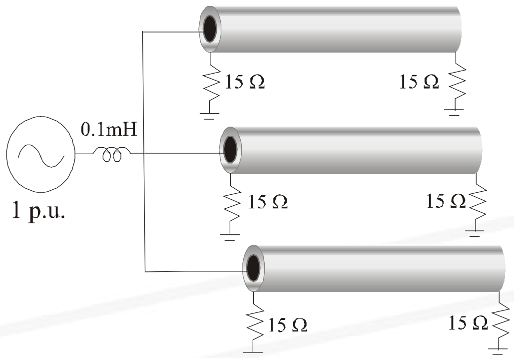


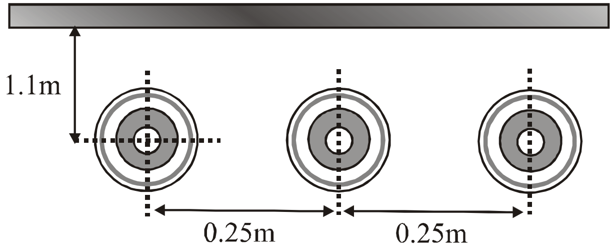
Figure 4. Current waveforms at load line–end.

Example 3: WB Model.

Simultaneous energizing of a *25 km* long cable. Inductive source with $L = 10^{-4}\ H$. Sheaths are grounded at both ends with grounding resistances of $R = 15\ \Omega$. Cores are open–ended at far end.



Cable system longitudinal view.



Transversal Geometry.

TABLE I
Cable's Electrical and Physical data

Core Inner radius	3.175mm
Core Outer radius	12.54mm
Sheath Inner radius	22.735mm
Sheath Outer radius	26.225mm
Insulation Outer radius	29.335mm
Core resistivity	$2.1\text{x}10^{-8}$ Ohm m
Sheath resistivity	$1.7\text{x}10^{-8}$ Ohm m
Earth resistivity	250 Ohm m
Earth relative permeability	1
Insulator relative permeability	1
Core insulator relative permittivity	3.5
Shield insulator relative permittivity	2.0
Insulation loss factor	0.001
Cable length	1km

Example 3: WB Model (continuation).

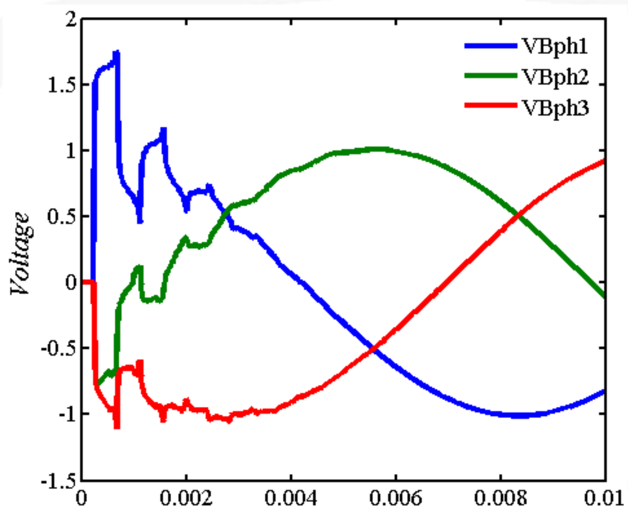


Fig. 1)

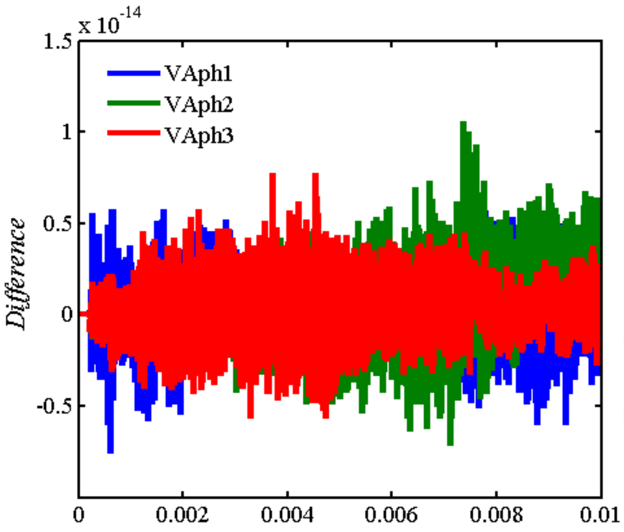


Fig. 2)

Fig. 1) **Nuclei Voltages at far end.**

Fig. 2) **Differences between new model and WB in EMTP-RV.**

Fig. 3) **Comparison with FD Line**

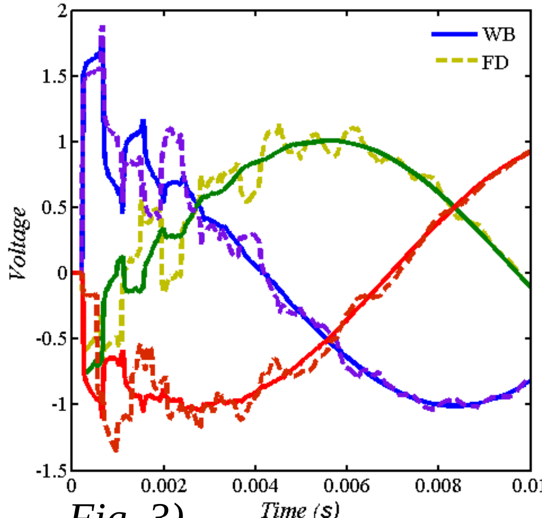


Fig. 3)

FD Real–Time Performance

Real-time performance of FD-line model on RT-LAB 2.4 GHz target

# of phase	# of pole for admittance function (all mode total)	# of pole for propagation function (all mode total)	Real-time time step of the complete model (μs)
3	11	6	**6**
3	39	67	**6**
6	15	21	**8**
6	71	134	**9**

Conclusions.

1. ***CP MODEL*** IS THE FASTEST, BUT HAS THE LOWEST ACCURACY. RECOMMENDED ONLY FOR LINES OUTSIDE THE NETWORK AREA OF INTEREST.

2. ***FD MODEL*** IS 40 % FASTER BUT LESS GENERAL THAN ***WB***. USE FOR AERIAL LINES THAT ARE SYMMETRIC OR NEAR SYMMETRIC. DOUBLE THREE–PH CIRCUIT LINES, ONLY IF AT THE SAME TOWERS.

3. ***WB MODEL*** CURRENTLY IS THE MOST GENERAL. IN CERTAIN CASES IT STILL PRESENTS COMPLICATIONS. THESE ARE CAUSED MOSTLY BY THE FITTERS.

4. NEW ***FD LINE*** AND ***WB LINE*** VERSIONS READY FOR OFF–LINE STUDIES IN MULTI–PROCESSOR SIMULATORS.

Conclusions.

***FD LINE* NEW VERSION:**

5. FULL STATE SPACE STRUCTURE, NODAL (NORTON) FORM INSTEAD OF ORIGINAL THEVENIN.
6. ACCEPTS EMTP PARAMETERS (BODE FITTING AND Z_C).
7. LOW ORDER MODELS OBTAINED THROUGH WIGHTED VECTOR FITTING (***WVF)***.
8. MODEL CAN BE SPLITTED FOR PARALLEL EXECUTION WITH MULTI–CORE PROCESSORS.

***WB LINE* NEW VERSION:**

9. NEW CODE OPTIMEZED (FULL ***SS*** STRUCTURE).
10. COMPLES STATES SOLVED WITH REAL ARITHMETIC.
11. NEW FITTER: ***WVF***.
12. WB MODEL CAN BE SPLITTED TOO.

ONGOING WORK: IMPLEMENTING REAL–TIME CODE FOR ***FD*** AND ***WB*** TYPE MODELS.